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A New Approach to the Generalized Transfer Function and Its Application to Biomedical Signal Analysis

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Abstract. In this paper, we present a method for modeling an electrocardiogram (ECG) signal using time-varying parameters by considering that the signal is generated by a linear, time-varying (LTV) system with a stationary white noise input, and then by estimating the time-varying coefficients of the LTV system. In effect, since the ECG signal is considered to be non-stationary, this method is based on the Wold-Cramer representation of a non-stationary signal. The relationship between the generalized transfer function of an LTV system and the time-varying coefficients of the difference equation of a discrete-time system is not addressed clearly in the literature; in this paper, we propose a new approach to this problem and apply it for modeling a human ECG signal. We first derive a relationship between the system generalized transfer function and the time-varying parameters of the system. Then we develop an algorithm to solve for the system time-varying parameters from the time-frequency kernel of the system output using the time-varying autocorrelation function (TVACF) and by modifying the modified least-square (MLS) and Durbin's approximation algorithms. A comparison between the proposed algorithm and the recursive least-square (RLS) and RLS lattice (RLSL) algorithms is considered. Computer simulations illustrating the effectiveness of our algorithm are presented when the signal is embedded in noise.

1. Introduction

The problem of signal modeling has attracted a considerable attention during the past few decades due to its large number of applications in diverse fields such as medicine [3, 7], where, the analysis and the modeling of a biomedical signal of practical importance. In this paper, we emphasize on the modeling problem, which is very important in the prediction and the compression of an electrocardiogram (ECG) signal [3,7,10], and [16]. An ECG signal, which is non-stationary, can be modeled by considering that it is generated by a linear, time-varying (LTV) system with a zero-mean, unit variance stationary white noise input. A subclass of discrete LTV systems is one for which the input and the output sequences *x (n)* and *y(n)* satisfy a LTV difference equation of the

Form

$$y(n) = \sum_{k=0}^{q} b_k(n)x(n-k) - \sum_{k=1}^{p} a_k(n)y(n-k) + \eta(n) \tag{1}$$

where $a_0(n)=1$, $\{a_k(n)\}$ and $\{b_k(n)\}$ are sets of time-varying parameters, $p \geq q$, $x(n)$ is a stationary signal and $\eta(n)$ is a white noise with symmetrical distribution. As in previous work, a convenient way to solve equation (1) is to approximate the time-varying coefficients of the system by constant parameters over each effective data window [12;15], which is an inherent assumption in the formulation of the recursive least-square (RLS) algorithms, for example. This assumption clearly limits the tracking ability and often introduces a bias in the parameter estimates. To alleviate this problem the system parameters are allowed to vary with time over the effective data window.

A simple approach is to replace the time-varying coefficients with their second-order expansion [15] or an arbitrary order expansion [12] as in the case of either the RLS polynomial lattice algorithms (RLSL). This assumption simplifies the time-varying identification problem in such a way that the equations resemble those in the time-invariant case. This is an interesting assumption; however, it has some pitfalls. The first one is that to approximate the time-varying coefficients we may need a large expansion order, depending on the type of polynomials used in the decomposition of the parameters. Also, there is no clear procedure to choose the polynomials *a-priori*. Moreover, the dimension of the problem increases as we increase the expansion order of the polynomials. In this paper, we propose a new method for estimating the time-varying parameters $\{a_k(n)\}$ and $\{b_k(n)\}$ without the above restrictions. The proposed method alleviates the above shortcomings by estimating the time-varying parameters at each instant of time, which is more general than both of the RLS and the RLSL approaches. The organization of this paper is as follows. In section 2, the necessary background material is presented. In section 3, we propose a relationship between the generalized transfer function of the LTV system and the time-varying coefficients of its difference equation. Based on this relationship, a new algorithm is proposed in section 4 for solving for the time-varying coefficients of the LTV system by modifying the well known modified least-square (MLS) method to fit to the time-varying case. In section 5, we present some numerical examples that show the performance of the algorithm through some distance measures between the actual and the estimated models, and then we apply the developed algorithm to the modeling of a human ECG signal.

3. Fundamental Relationship

In this paper, we shall consider a zero-mean discrete-time stochastic processes. If the process is stationary, it possesses a decomposition called the Wold decomposition, which states that the process $y(n)$ can be obtained as the output of a causal linear filter driven by a white noise *x(n)*. This filter has an infinite impulse response and therefore is not suitable for a parsimonious representation of the process. For this reason it is more convenient to use the autoregressive moving-average (ARMA) model which consists of a finite number of parameters and can be represented as

$$y(n)+a_1 y(n-1)+\ldots+a_p y(n-p)=b_0 x(n)+\ldots+b_q x(n-q) \tag{2}$$

The Wold decomposition is more general than the ARMA model which only applies to processes with rational spectra. To have a one-to-one correspondence between ARMA models and rational spectra, it is necessary to stipulate additional constraints on the parameters a_i and b_i:

1) The denominator polynomial $A(z)=1+a_1z^{-1}+\ldots+a_pz^{-p}$ must have all its roots inside the unit-circle to insure stability.
2) The numerator polynomial $B(z)=b_0+b_1z^{-1}+\ldots+b_pz^{-q}$ must have all its roots inside the unit-circle to have non-minimum phase system, and therefore invertible.

Since the variance of the input signal *x(n)* has been normalized to be 1, the rational spectrum $S(\omega)$ for *y(n)* is

$$S(\omega)=\left|\frac{B(z).B(z^{-1})}{A(z).A(z^{-1})}\right|_{z=e^{jw}}$$

Let us now assume that the process *y(n)* be non-stationary. Cramer [6] showed that it still possesses a Wold decomposition in terms of its innovation *x(n)* and its generating system. However, the linear system generating *y(n)* is no longer time-invariant when driven by the innovation *x(n);* the impulse response of this system will be a time-dependent function so that

$$y(n)=\sum_{m=0}^{\infty}h(n,m)x(n-m)=\sum_{m=\infty}^{n}h(n,n-m)x(m) \tag{3}$$

It is possible to obtain a finite-order representation of the process, e.g., an equivalent time-varying ARMA (TV-ARMA) model of the form

$$y(n)+\ldots+a_p(n-p)y(n-p)=b_0(n)x(n)+\ldots+b_q(n-q)x(n-q) \tag{4}$$

The input-output relationship in equation (3) may be realized as an ARMA recursion like equation (4) if and only if there exists an integer *n*, and functions $a_1(n),\ldots, a_k(n)$ such that [6]

$$h_i(n) + z^{-1}(a_1(n)h_{i-1}(n)) + \ldots + z^{-k}(a_k(n)h_{i-k}(n)) = 0 \qquad \forall i \geq k \tag{5}$$

where the operator z^{-1} represents the delay operator acting on the set of real-valued functions defined over the ring ***Z*** of integers, so that $z^{-1}\{f(n)\}=f(n-1)$. The parameters of the AR part in (4) are obtained precisely as these functions $a_1(n),\ldots, a_k(n)$.

Another answer to the question of existence of a time-varying ARMA model was given by Huang and Aggarwal [9] which worked on the covariance $H(n,m)$ of the process $y(n)$ $H(n,m)=E\{y(n)y(m)\}$. The process $y(n)$ may be realized by an ARMA model driven by a white noise, like (4), if and only if there exists an integer k and functions $\chi_i(n)$, $\psi_i(n)$ such that

$$h(n,m) = \sum_{i=1}^{k} \chi_i(n)\psi_i(m) \tag{6}$$

Covariances having property (6) are called "*degenerate*" and are often encountered in the literature under the name "*separable*." The existence of a time-varying ARMA(k,k-1) representation for a non-stationary process is ensured by the theorems given in [6;9]. The uniqueness of the representation is obtained by constraining the ARMA model to be invertible, but this leads to conditions on the time-varying impulse response $\{h(n,m)\}$ and its inverse, which cannot be easily expressed in terms of the coefficients $a_i(n)$ and $b_i(n)$. Now, let us discuss the problem of estimating the time-varying coefficients $a_i(n)$ and $b_i(n)$. In signal processing, one usually has only one trajectory of the stochastic process $y(n)$, i.e., a set of observations $\{y_0, \ldots, y_T\}$. Apart from likelihood techniques working on the samples themselves, most estimation techniques use some kind of estimate of the covariance function of the process. If the process is stationary, one uses an ergodic estimator. When $y(n)$ is a non-stationary process, this approach is no longer feasible, so that, without further assumption, it is not possible to identify a time-dependent model.

The simplest assumption which can be made is that the process $y(n)$ is not too far from stationary, so that the variation of the time-varying coefficient $a_i(n)$, being rather smooth, can be tracked by adaptive algorithms (least-square, gradient, etc.). This is also the assumption implicitly made in the sliding DFT. The tracking ability of these algorithms is prescribed by either size of a window or the value of a fading rate, and there lies the limitation of these methods [4]: if the process evolves too quickly, the algorithm will not properly track the evolution of the coefficients, unless the window becomes very short, which in turn degrades the spectral resolution.

Another assumption might be that the coefficients evolve in a Markovian way; the coefficients $a_i(n)$ and $b_i(n)$ are considered stochastic, and the output of a linear finite-order system driven by a white noise. If the parameters of this Markov model are known, the estimation of the a_i's and b_i's is simply the estimation of the state of the Markov model, a problem that can be solved with the help of the Kalman filter. If the transition matrix and the noise covariance are unknown, the problem becomes an intricate nonlinear problem of simultaneous state and parameter estimation.

A third assumption is that the coefficients $a_i(n)$ and $b_i(n)$ may be approximated satisfactorily by a weighted combination of a small number of known functions. This idea seems to have been used for the first time by Rao [15] who replaced the time-dependent coefficients with their second-order expansion

$$a_i(n) = a_{i0} + n a_{i1} + \frac{n^2}{2} a_{i2} \tag{7}$$

The set of known functions is here restricted to the functions

$$f_0(n)=1; \quad f_1(n)=n; \quad f_2(n)=\frac{n^2}{2}$$

Now, we present an algorithm to identify the time-varying parameters of an LTV system from its generalized transfer function. As already mentioned, this algorithm can be used in modeling a non-stationary signal by considering the signal as the output of an LTV system with a stationary white noise input. If the variation of the time-varying parameters is much slower than the variation of the process $y(n)$, which is the case when $x(n)$ is white noise, then taking the z-transform of both sides of equation (1) yields

$$[\sum_{k=0}^{p} a_k(n) z^{-k}] Y(z) = [\sum_{k=0}^{q} b_k(n) z^{-k}] X(z) \tag{8}$$

where $z=e^{j\omega}$. Since $y(n)$ is non-stationary, for a unit-impulse input, equation (8) becomes

$$[\sum_{k=0}^{p} a_k(n) z^{-k}] \Gamma_y(n,z) = \sum_{k=0}^{q} b_k(n) z^{-k},$$

or

$$\Gamma_y(n,z) = \frac{B(n,z)}{A(n,z)} = \frac{\sum_{k=0}^{q} b_k(n) z^{-k}}{\sum_{k=0}^{p} a_k(n) z^{-k}} \tag{9}$$

where $\Gamma_y(n,z)$ represents the time-frequency kernel of the process $y(n)$ expressed at each instant of time n, and since x(n) is a white-noise process, the time-frequency kernel

$\Gamma_x(n,z)=1$ for all n. Also, $\Gamma_y(n,z)$ represents the generalized transfer function of the LTV system with $\{a_k(n)\}$ and $\{b_k(n)\}$ representing the time-varying coefficients of its denominator and numerator, respectively.

The time-varying system impulse response $\zeta_y(n,m)$ can be estimated from the time-varying frequency response $\Gamma_y(n,\omega)$ using any time-frequency kernel estimator. Then the time-varying coefficients of the LTV system are obtained from the inverse Fourier transform of the estimate of $\Gamma_y(n,\omega)$. The modified least-square (MLS) algorithm [13] can then be used to solve for the time-varying coefficients of the LTV system after modifying it to fit for LTV system identification problem. If we have *a-priori* knowledge of p and q, then the Prony approximation algorithm [6] can also be used to solve for $\{a_k(n)\}$ and $\{b_k(n)\}$. In the MLS, our objective is to obtain a rational function which approximately has $\zeta_y(n,m)$ as its impulse response, by solving the MLS minimization problem.

4. Estimation of the time-varying ARMA coefficients

This section is devoted to estimation of the time-varying coefficients of the model. From equation (9), we have

$$\Gamma_y(n, e^{j\omega_k})\, A(n, e^{j\omega_k}) = B(n, e^{j\omega_k}) \tag{10}$$

where $\omega_k=2\pi k/N$, N being the length of the available data points for $k=0,1,..,N-1$. For convenience, let us replace $e^{j\omega_k}$ by z_k. In the following, we propose a procedure for finding the parameters of a rational function that approximates equation (9) by minimizing the following quadratic error

$$\varepsilon^2(n) = \sum_{k=0}^{N-1} |\Gamma_y(n,z_k)A(n,z_k) - B(n,z_k)|^2$$

The solution of this problem can be obtained by recursively solving the system of time-dependent linear equations [13;8]

$$K_n[1, a_1(n), \ldots, a_p(n)]^t = [\gamma_p(n), 0, \ldots, 0]^t \tag{11}$$

In (11), $\gamma_p(n)$ is the time-varying minimization error for $A(n,z_k)$ of order p. K_n is a $(p+1)$x$(p+1)$ time-dependent matrix with entries given by

$$K_n(i,j) = r_n(|i-j|) - \sum_{m=0}^{q-p+\min(i,j)} \zeta_y(n,m)\zeta_y^*(n,m+|i-j|)$$

where $i,j=0,1,...,p$, $\zeta_y(n,m)$is the impulse response sequence of the LTV system or the Fourier coefficients of the generalized transfer function $\Gamma_y(n,z)$ and $r_n(.)$ is the TVACF of $y(n)$. Once the time-dependent coefficients $\{a_k(n)\}$ are calculated by using equation (11), the time-varying MA coefficients $\{b_k(n)\}$ can be estimated as

$$b_k(n)=\sum_{j=0}^{p} a_j(n)\zeta(n,k-j) \tag{12}$$

where $k=0,1,...,q$.

In (12), $b_k(n)$ depends on the time-varying response $\zeta(n,m)$, though we should look for better and more robust procedure for estimating the MA parameters. Therefore, consider the following procedure for estimating the time-varying MA parameters. After obtaining the time-varying AR parameters, we can form an equivalent time-varying MA model by removing the time-varying AR component to get the new MA process

$$g(n) = y(n) - \sum_{i=1}^{p} a_i(n)y(n-i).$$

This can be considered as the output of an equivalent time-varying MA model represented by

$$g(n)=\sum_{i=0}^{q} b_i(n-i)x(n-i).$$

From the above, the TVACF of $g(n)$ can be expressed as

$$r_n^g(m)=\sum_{i=0}^{q}\sum_{j=0}^{q} b_i^*(n-i)b_{m+j}(n-j) \tag{13}$$

where $m=0,1,...,q$. The problem now is to estimate $\{b_i(n)\}$ from the TVACF of $g(n)$. Equation (13) is nonlinear and, in general, does not have a unique solution. However, at each instant of time n, $\{b_k(n)\}$ can be estimated by modifying the Durbin's approximation algorithm for the estimation of the parameters of an MA(q) process which is summarized as follows:

1. Using the data sequence $\{g(0), g(1), ..., g(N-1)\}$, fit a large order time-varying AR model using the autocorrelation method. For an AR model order L, where $q<<L<<N$, the white noise variance estimator σ^2_n is given by

$$\sigma_n^2 = \tilde{r}_n^g(0) + \sum_{k=1}^{L} \tilde{a}_n(k)\tilde{r}_n^g(k) \tag{14}$$

2. Using the AR parameter estimates obtained from above as the data, use the autocorrelation method with order q to find

$$\tilde{b}_n = \{\tilde{b}_n(1), \tilde{b}_n(2), \ldots, \tilde{b}_n(L)\}.$$

That is

$$\tilde{b}_n = -\tilde{R}_n^{-1}\tilde{r}_n \tag{15}$$

where

$$[\tilde{R}_n]_{ij} = \frac{1}{L+1}\sum_{k=0}^{L-|i-j|}\tilde{a}_n^*(k)\tilde{a}_n(k+|i-j|) \tag{16}$$

and

$$[\tilde{r}_n]_i = \frac{1}{L+1}\sum_{k=0}^{L-1}\tilde{a}_n^*(k)\tilde{a}_n(k+i) \tag{17}$$

It should be noticed also that the model orders, p and q, may vary with time. This means that at time n_0 we may need p_0 and q_0 to model the signal, however, at time n_d, for example, we may need p_d and q_d in order to model that signal. In this paper, this assumption is not considered. The accuracy of the estimated time-varying coefficients depends on the correct estimate of $\Gamma_y(n,\omega)$. If p and q are not known *a-priori*, one can use any order estimation algorithm such as the ones addressed in [1,4,5].

5. Experimental Results

The performance of our method is evaluated through distance measures defined in terms of the original model and the estimated one. One way of examining the closeness of fit of $\{a_k(n)\}$ and its estimate is to evaluate the error measure E_i, where

$$E_i = \frac{A_i}{B_i} \tag{18}$$

In (18), A_i denotes the power of the estimated error and is given by

$$A_i = \sum_n |a_i(n) - \beta \hat{a}_i(n)|^2,$$

Where as

$$B_i = \sum_n |a_i(n)|^2$$

The error A_i is minimized by choosing

$$\beta = \frac{\sum_n a_i(n)\hat{a}_i(n)}{\sum_n \hat{a}_i^2(n)}.$$

That is, we look for the best constant β to make $a_i(n)$ and its estimate comparable. In addition to this distance measure, we will also use the signal-to-noise ratio (SNR) defined as

$$\mathrm{SNR} = 10\log_{10}\frac{\sum_n |y(n)|^2}{\sum_n |y(n) - \hat{y}(n)|^2}$$

The mean-square error (MSE) between the actual signal and its estimate is also considered. To illustrate the performance of the time-varying algorithm proposed in the previous section, we first apply it to a system that has two components of which one is time-invariant and the other is time-varying, with and without noise; we then apply it in modeling a human ECG signal.

Example 1:
Let us have a non-stationary signal generated from a second-order time-varying autoregressive (TVAR) model such that $p=2$, $q=0$ in equation (1), i.e.,
Where

$$y(n) = x(n) - a_1(n)y(n-1) - a_2(n)y(n-2)$$

$$a_1(n) = \begin{cases} 1 & 0 \le n < 32, 64 \le n < 96, 124 \le n < 160, 192 \le n < 228 \\ 0 & \text{otherwise} \end{cases}$$

$$a_2(n) = 2\left(\frac{n}{N} - \left(\frac{n}{N}\right)^2 + \left(\frac{n}{N}\right)^3 - \left(\frac{n}{N}\right)^6\right)$$

for n=$0,1,\ldots,N-1$, $N=256$. We assume then there is no noise, i.e., $\eta(n)=0$ in equation (1), and then the input $x(n)$ is a zero-mean, unit-variance stationary white noise. Using 60 Monte-Carlos, the true and the estimated coefficients of the LTV system are shown in Fig. 1 and 2 for the estimates of $a_1(n)$ and $a_2(n)$, respectively. The results of the error measures are shown in Table 1, and the original signal and the estimated one are shown in Fig. 3, with MSE shown in Fig. 4. We observe from the figures that the estimated coefficient of the time-invariant parameter is close to the actual one and then the estimated coefficient of the time-varying parameter is tracking the actual one. Also, we observe that the MSE between the estimated signal and the actual one is indeed small.

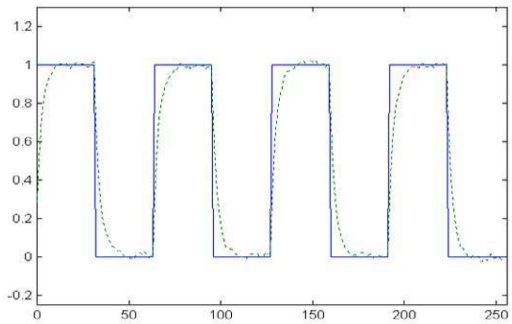

Fig. 1. Plots of *$a_1(n)$* (solid) and its estimate (dotted) of example 1.

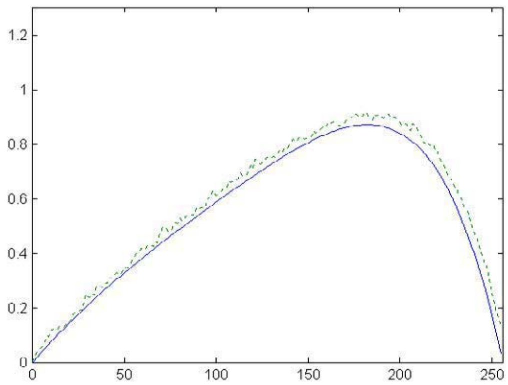

Fig. 2. Plots of *$a_2(n)$* (solid) and its estimate (dotted) of example 1.

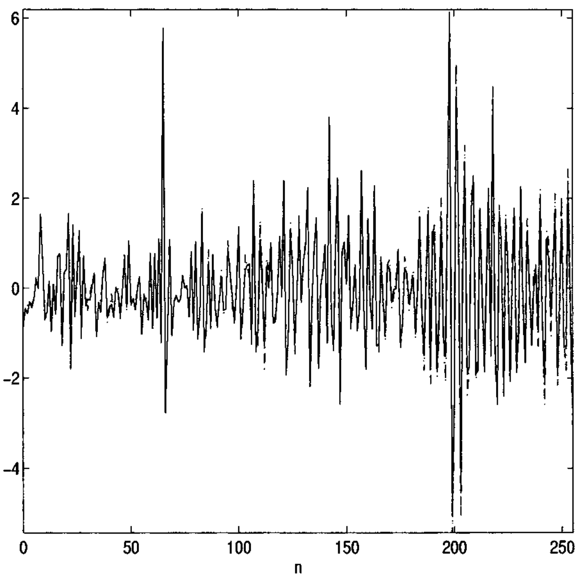

Fig. 3. Plots of *y(n)* (solid) and its estimate (dotted) of example 1.

Example 2:

Let the signal *y(n)* given in example 1 be now embedded in a stationary Gaussian noise with SNR=5 dB. Computed from 60 Monte-Carlos, the true and estimated coefficients of the LTV system are shown in Figures 4 and 5 for the estimates of $a_1(n)$ and $a_2(n)$, respectively. The results of the error measures are shown in Table 1, and the original signal and the estimated one are shown in Fig. 6. We observe here the effect of noise in degrading the estimation of $a_1(n)$; however, it has less effect on the time-varying parameter estimate of $a_2(n)$. Also, we observe that the MSE is not significant except for large values of n, $200<n<N$.

Table 1. The estimated time-varying coefficients of $\zeta_y(n,m)$

Figures	E_1	E_2	SNR(dB)
2,3,4,5	0.0011	0.0237	16.2354
6,7,8,9	0.0168	0.0215	7.8897

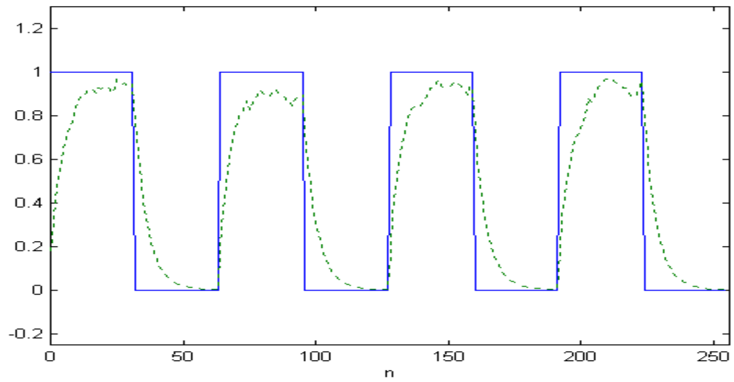

Fig. 4. Plots of $a_1(n)$ (solid) and its estimate (dotted) of Example 2.

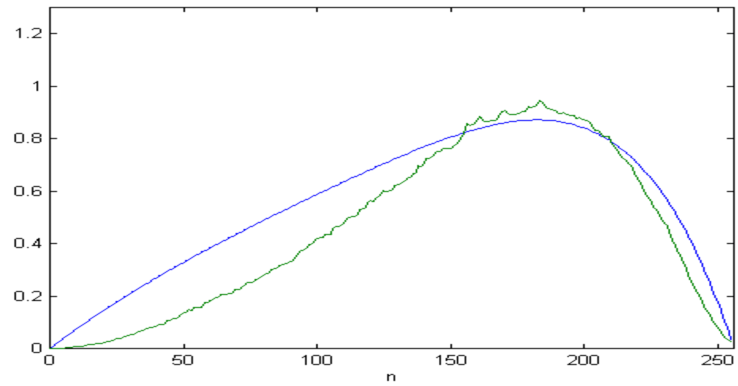

Fig. 5. Plots of $a_2(n)$ (solid) and its estimate (dotted) of example 2.

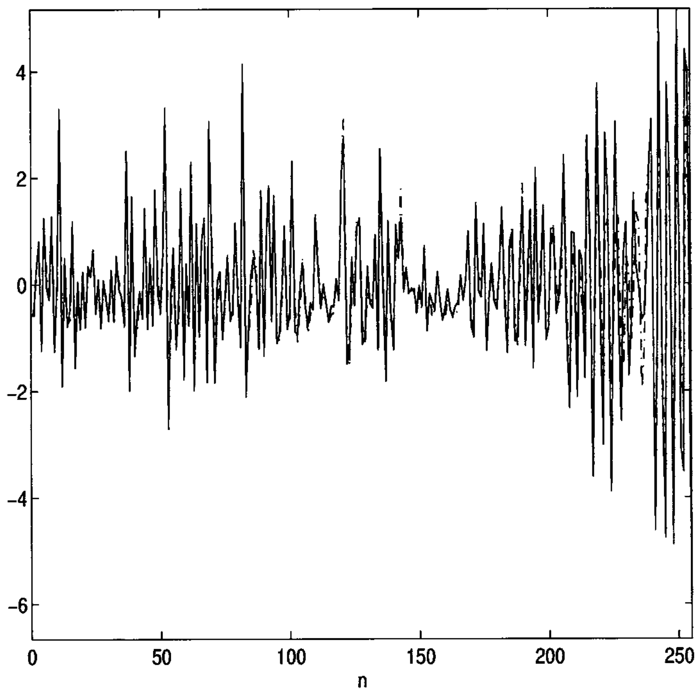

Fig. 6. Plots of *y(n)* (solid) and its estimate (dotted) of example 2.

Example 3:

In this example, we consider the modeling of 512 samples of a human ECG signal with warm water in ear. The zero-mean, actual ECG signal is shown in Fig. 7. The synthesized ECG signal versus the actual ECG signal is shown in Fig. 8, which shows that the algorithm is adaptive with time.

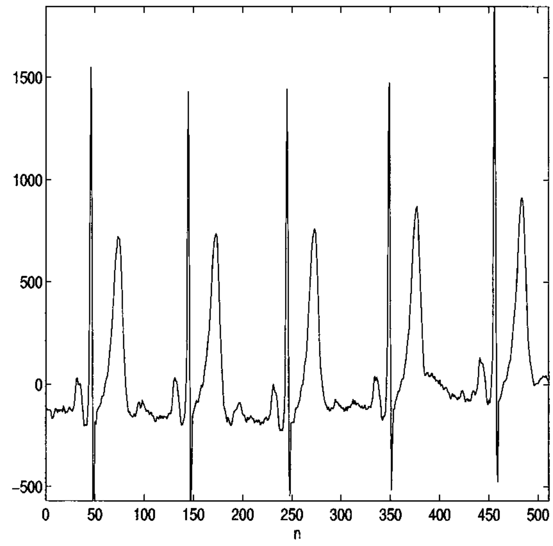

Fig. 7. ECG signal of example 3.

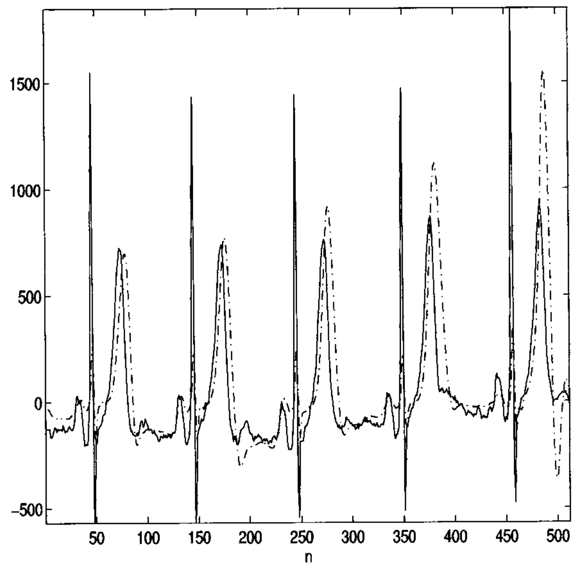

Fig. 8. Part of the ECG signal (solid) with its estimate (dotted) of example 3.

6. Conclusion

In this paper, the problem of identifying the parameters of an LTV ARMA model was addressed. A new approach for estimating the time-varying parameters of a non-stationary signal is presented. Also, a modification of the MLS method for solving for the time-varying parameters was proposed. It has been demonstrated through computer simulations that the proposed approach performs well in modeling non-stationary signals generated by a time-varying autoregressive model with a white noise input. Computer simulations illustrating the effectiveness of our algorithm are presented when the signal is embedded in noise. The proposed algorithm was applied for the modeling of a human ECG signal.

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اتجاه جديد إلى دالة النظام العامة وتطبيقه على تحليل الإشارات الطبية

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ملخص البحث. هذا البحث يقترح اتجاه رياضي جديد إلى دالة النظام العامة المتغيرة مع الوقت واستخدام هذه الطريقة لتحليل ونمذجة الإشارات الطبية الصادرة من القلب، أو ما تسمى إشارة (ECG). وحيث أن خصائص هذه الإشارة تتغير مع الوقت، لذا فإننا في هذه الطريقة نفرض أن إشارة القلب صادرة من نظام فيزيائي متغير مع الوقت، أو ما يسمى (LTV)، وأن الإشارة الداخلة إلى هذا النظام هي عبارة عن ضوضاء من نوع (white noise).

ونظرا لأن العلاقة الرياضية بين دالة النظام المتغير مع الوقت وبين المعادلات التفاضلية لهذا للنظام ليست مبوبة بوضوح في البحوث السابقة لهذا البحث، فإننا في هذا البحث نقترح طريقة جديدة، أكثر وضوحا وسهولة، تمثل هذه العلاقة مع إثباتها رياضيا وتطبيقها على نمذجة الإشارات الطبية الصادرة من القلب.

أيضا هذا البحث يقترح طريقة لإيجاد متغيرات مقام الدالة وطريقتين لإيجاد بسط الدالة رياضيا. علاوة على ذلك، فقد تم مناقشة بعض الطرق الأخرى المنشورة في هذا الصدد وتم إيضاح الفروق بينها وبين الطريقة المقترحة. في نهاية هذا البحث تم الحصول على بعض النتائج الرقمية والبيانية لإثبات نجاح الطريقة المقترحة في النمذجة.